SCDC-2049

DTIE MICROCARD ISSUANCE DATE

1/20 1964

NOV 26 1964

CONF-319-1 E2

MASTER

MR. P. T. SCHOENEMANN - 9120

Re: Review of Material Submitted for Publication or Presentation

Enclosures: (a) Transmittal memorandum, dated November 1, 1963, P. T. Schoenemann to W. Jenkins, 8233-1.

Analyzing Nonstationary Vibration Data

Your paper has been reviewed. Our review concurs that the paper is unclassified and that its release to the public would have no adverse effect upon the common defense and security. It is released under Topic 1013 of the Sandia Corporation Classification Handbook, an AEC-approved guide.

CLASSIFICATION REPRESENTATIVE

Telecon approval 340 WAS

Your paper has been reviewed. There is no objection from a patent standpoint to its release to the public.

Telecon approval WAS
INVENTION REPORTS,

In consideration of the above endorsements the release of your paper, SCDC 2049, Techniques for Analyzing Nonstationary Vibration Data, is approved for presentation at the 33rd Symposium on Shock, Vibration, and Associated Environments, in Washington, D.C. December 3-5, 1963.

D. R. COTTER - 9100

Facsimile Price $ 1.60
Microfilm Price $ .80

Available from the
Office of Technical Services
Department of Commerce
Washington 25, D. C.

20011010 153

SCDC-2049

Techniques for Analyzing Nonstationary Vibration Data

by

P. T. Schoenemann
Sandia Corporation, Livermore Laboratory

November 1963

Techniques for Analyzing Nonstationary Vibration Data

I. Introduction

In most instances the in-flight vibration environment endured by a missile warhead is a nonstationary random process. However, most current methods of random vibration data analysis assume that the data represents stationary, ergodic random phenomena, which is generally true only for laboratory shaker tests and most shipping environments. If a process is nonstationary, the standard methods do not apply with assurance, and the resulting analysis must be treated with care.

Considerable theoretical work on nonstationary random phenomena has been published during the last decade. Little of this theory has been applied to the task of analyzing missile flight vibration data other than assuming that the data is piecewise stationary and then treating each piece in the conventional manner. However, it is possible and sometimes quite practical to treat the data as nonstationary to derive a more meaningful representation of the actual environment.

II. Theoretical Background

Various investigators (Refs. 1, 2, 3, 4, 7, 8, 16) have developed expressions which describe the spectrum of a nonstationary process as a function of both time and frequency. Some of their relationships are particularly applicable to missile vibration analyses. Although many of the developments differ in detail, a common factor to the various theoretical treatments is a time varying expression for the power spectrum, $P(t,f)$, where f is frequency in CPS and t is time in seconds.

These expressions are derived in the following general manner.

Consider a general signal $s(t)$.

The correlation function is defined as usual by

$$R(t,\tau) = E\left[s(t)s(t+\tau)\right] \tag{1}$$

τ = Tau

where E represents the mathematical expectation.

For stationary, ergodic processes $R(t,\tau)$ is constant with t so it can be represented as a function of τ only,

$$R(t,\tau) = R(\tau) \tag{2}$$

and the familiar manipulations can be made (Ref. 8) to obtain the power spectrum.

$$P(f) = \int_{-\infty}^{\infty} R(\tau)e^{-j2\pi f\tau}d\tau = 2\int_{0}^{\infty} R(\tau)\cos 2\pi f\tau d\tau \tag{3}$$

$P(f)$ is the Fourier transform of $R(\tau)$.

For the nonstationary case, a parallel expression can be obtained.

$$p(t,f) = 2\int_{0}^{\infty} R(t,\tau)\cos 2\pi f\tau d\tau \tag{4}$$

This is the spectrum of the instantaneous power of $s(t)$, as shown in References 1 and 8. There is, in general, a different spectrum for every instant t.

This expression for $p(t,f)$ is for only one sample function of the random process. Other sample functions will yield, in general, different functions for $p(t,f)$. For stationary ergodic processes one sample function is statistically representative of all sample functions <u>and</u> one time interval of the

sample function is statistically representative of the whole sample function, so the data analysis problem is greatly eased. This, unfortunately, is not the case for nonstationary processes.

The vibration environment of a missile flight is by no means stationary, but it is essentially repeatable from flight to flight for a given set of launch and re-entry parameters. Over the ensemble of similar missile flights, averaging yields an estimate for the power spectrum of the nonstationary process at a given time t.

$$E\left[p(t,f)\right] = \frac{1}{n}\sum_{i=1}^{n} p_i(t,f) = \overline{p(t,f)} \tag{5}$$

where p_i is the spectrum of the i^{th} flight,

n is the number of flights.

$\overline{p(t,f)}$ then is the expected spectrum of the instantaneous power at time t. The desired quantity, however, is the expected spectrum of the power averaged over some interval T.

$$P(t,f) = \frac{1}{T}\int_{t-T}^{t} \overline{p(t,f)}\, dt \tag{6}$$

The length of the averaging interval T affects the value of $P(t,f)$ at any time t since it determines the degree of time smoothing afforded to the spectrum. A correct choice of T depends upon engineering compromises between the degree of time resolution desired, the response characteristics of the system under vibration, and the severity of the nonstationarity.

It is shown in Reference 1 that the time averaging and statistical averaging can be performed in any order. This is important for data analysis where time

averaging on a single flight is usually performed before all the flights are averaged together. In mathematical terms

$$P(t,f) = \frac{1}{T}\int_{t-T}^{t} \frac{1}{n}\sum_{i=1}^{n} p_i(t,f)dt = \frac{1}{Tn}\sum_{i=1}^{n}\int_{t-T}^{t} p_i(t,f)dt \tag{7}$$

III. Separable Random Processes

There are some specialized forms of nonstationary processes of frequent practical interest that have been studied by several investigators (Refs. 6, 8, 11, 12). Most of these fall into the so-called "separable class" where the correlation function can be represented as

$$R(t,\tau) = R_1(x)R_2(y) \tag{8}$$

where x and y are related in some specific way to t and τ.

Of particular interest are signals separable with respect to t and τ.

$$R(t,\tau) = R_1(t)R_2(\tau) \tag{9}$$

($R_1(t)$ is constant for wide-sense stationary processes.) For many practical situations (Refs. 8 and 12) a good approximation to the physical situation is

$$R(t,\tau) \cong R_1(t)R_2(\tau)$$

where $R_1(t)$ is proportional to the instantaneous power.

This is particularly applicable in the case where some wide-sense stationary random signal is slowly amplitude-modulated. The power spectrum

can then be approximated by

$$P(t,f) \cong R_1(t)P_2(f) \tag{10}$$

where $P_2(f)$ is the Fourier transform of $R_2(\tau)$.

In many cases, even though a nonstationary process is not strictly separable, the assumption of separability yields useful approximations.

IV. Application

Various practical problems arise in the application of the nonstationary theory. Most important of these are:

1. The complexity of the equipment needed to perform the necessary mathematical operations.
2. The assigning of statistical meaning to data samples taken from the nonstationary environment.
3. The interpretation of the resulting data for use in component or system design and for developing meaningful laboratory environmental test specifications.

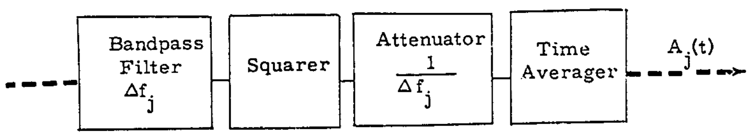
In some cases there are no practical solutions for all these problems. However, there are many physical situations where useful information can be obtained.

The experimental determination of $P(t,f)$ at a point on the (t,f) plane is impossible (Ref. 8) since only averages are considered. The average value over an area $\Delta t\,\Delta f$ can be estimated by taking a time average over some frequency band.

Equipment suitable for the spectral analysis of nonstationary random signals must measure all the spectrum of interest all of the time. The

analysis can be performed on a digital computer but this is a formidable task even for a computer with a large memory. However, for installations with large computing facilities, computer time may be less expensive than the installation and operation of some special-purpose equipment.

One practical method of estimating P(t,f) is to use a set of comb filters with contiguous frequency bands to separate the data in the frequency domain and then use squaring and appropriate averaging on the output of each filter to produce real-time analysis. A generalized block diagram of one channel of such a device is shown in Figure 1.



One Channel of a Multiple Channel Analyzer

Figure 1

The output of each channel can then be represented as a time varying function $A_j(t)$, which represents the squared signal distributed over some frequency band Δf_j and averaged over some time interval Δt.

The multiple filter device similar to Figure 1 but with an integrator for a time averager is described in Ref. 13. The device does not yield a continuous $A_j(t)$ for each channel but gives samples every n seconds, where n is variable in discrete steps from 1 to 10 seconds. A device yielding a continuous $A_j(t)$ would utilize a low-pass filter for the time averaged. Fano (Ref. 7) examines the case for RC filters. An RC filter gives an exponentially-weighted time average. To perform the averaging indicated in Equation 6 requires com-

plex circuitry using delay lines to produce the transfer function

$$\frac{1 - e^{-sT}}{sT}$$

where s is the Laplace variable.

Such transfer function realization is uneconomical at present for multichannel real-time analog analyzers.

The output of each analyzer channel can be represented as a time varying function $A_j(t)$, which represents the squared signal averaged over some finite time interval T. Now, if we have data from n similar flights, the representation of P(f,t) becomes

$$P(f,t) = \frac{1}{n} \sum_{i=1}^{n} A_{ij}(t) \qquad \text{All } t, f \text{ in } \Delta f_j \tag{11}$$

where $A_{ij}(t)$ is the output of the j^{th} channel for the i^{th} flight.

In order to analyze the statistics of the collection of $A_{ij}(t)$ we must consider the ensemble at some $t = t_o$, a constant. $P(f,t_o)$ is the expected value for any given flight datum at t_o. The variance for the ensemble is estimated by

$$\sigma_j^2 = \frac{E \sum_{i=1}^{n} \left[A_{ij}(t_o) - P(f,t_o)\right]^2}{n - 1} \tag{12}$$

which is the formula for the unbiased estimate of the variance of a random variable. This means that the $A_{ij}(t_o)$ for the $(n + 1)^{th}$ flight will have an expected value estimated by Equation (11) and a variance estimated by Equation (12).

If s(t) is a Gaussian variable at all t_o, the output of the averaging circuits will be something between chi-square and Gaussian depending on the averaging

time T. Usually the determination of the distribution is done empirically. The determination of the distribution of the output of a linear system for non-Gaussian inputs is covered in a paper by Mazelsky (Ref. 17). In practice, the empirical approach is easier.

$P(f,t)$ as defined will be a variable for stationary processes as well as non-stationary processes. However

$$\lim_{n\to\infty} P(f,t_1) = \lim_{n\to\infty} P(f,t_2)$$

holds for stationary cases but not for non-stationary cases. The statistics for the stationary case have been treated thoroughly in the literature. As a measure of non-stationarity, a statistical confidence test can be made on a succession of time samples.

The third problem is to develop a sensible environmental test specification for electrodynamic shaker systems. The average environment can, with a lot of expense and complication, be represented by some artificial non-stationary process. However, most, if not all, testing laboratories utilize a stationary process for environmental simulation. The problem, then, is to derive a stationary process that in some meaningful way represents the non-stationary process of the missile environment. One method is to construct a composite spectrum with the level in each frequency band Δf_j greater than some arbitrary percentage of the $A_{ij}(t)$. This can be in general an overtest since maxima in the different frequency bands do not necessarily coincide in time. However such a specification does assure that each frequency band is tested to a significant level.

Often in practice this method of deriving a specification is not an overtest. When the environment can be closely approximated by a separable process

described by Equation 10

$$P(t,f) = R_1(t)P_2(f)$$

then the test specification can be

$$P(f) = \left[\text{Max } R_1(t)\right] P_2(f) \tag{13}$$

The RMS value of the test environment will then coincide with the maximum of the real environment.

One method of reducing the overtest if Equation (10) does not hold is to break the environment into sections where separate tests can be specified. For example, if the launch phase has predominantly low frequencies and the re-entry phase predominantly high frequencies, two separate tests can be run. This method increases testing time and costs, but may be worthwhile if appreciable component weight savings are affected.

V. Conclusions

Methods are available for the analysis of non-stationary vibration data. The resulting analysis is more voluminous than that for stationary environments since two variables, frequency and time, are important, rather than just frequency. For many missile flight environments where the statistics are time variable but repeatable, a stationary test specification can logically be derived from the non-stationary environment.

8

REFERENCES

1. Kharkevich, A. A., Spectra and Analysis (translated from Russian), Consultants Bureau, New York, 1960.
2. Page, C. H., Instantaneous Power Spectra, J. Applied Physics, Vol. 23, No. 1, January 1952.
3. Turner, C. H. M., On the Concept of an Instantaneous Power Spectrum and its Relationships to the Autocorrelation Function, J. Applied Physics, Vol. 25, No. 11, November 1954.
4. Lampard, P. G , Generalization of the Wiener-Khintchine Theorum to Nonstationary Processes, J. Applied Physics, Vol. 25, No. 6, June 1954.
5. Lampard, P. G., The Response of Linear Networks to Suddenly Applied Stationary Random Noise, IRE Trans. Circuit Theory, Vol. CT-2, March 1955.
6. Nuttall, A. H., Theory and Application of the Separable Class of Random Processes, MIT Res. Lab or Elect., Rept. 343, May 1958.
7. Fano, R. M., Short-Time Autocorrelation Functions and Power Spectra, J. of the Acoustical Soc. of Am., Vol. 22, No. 5, September 1950.
8. Zheleznov, N. A., Some Problems of the Spectral Correlation Theory of Nonstationary Signals, Radiotekhnika I Electronika, Vol. 4, No. 3, p. 359-379, 1959.
9. Nuttal, A. H., Theory and Application of the Separable Class of Random Processes, MIT Res. Lab. of Electronics, Tech. Report 346, May 26, 1958.
10. Borford, T. M., Nonstationary Velocity Estimation, Bell Syst. Tech. Journ., Vol. 37, No. 4, July 1958.
11. Silverman, R. A., Locally Stationary Random Processes, IRE Trans. on Information Th., IT3, No. 3, September 1957.

12. Gudzenko, L. I., On Periodic Nonstationary Processes, Radio Engineering and Electronics, Vol. 4, No. 6, 1959, p. 220-224.
13. Schoenemann, P. T., Real Time Analysis of Random Vibration Power Density Spectra, OSD/R&E, Bulletin No. 31, Shock Vibration and Associated Environments, April 1963, Part III.
14. Schoenemann, P. T., Measurement of Power Spectra for Nonstationary Random Signals, Sandia Corp. SCTM 311-62-81, April 1962.
15. Batkov, A. M., Generalization of the Shaping Filter Method to Include Nonstationary Random Processes, Automation & Remote Control (USSR), Vol. 20, #3, 1049-1062, 1959.
16. Kampe de Feriet, J. and Frenkiel, F. N., Estimation de la Correlation d'une Function Aleatoire non Stationnaire, Comptes Rendus, Acad. Sciences, Paris, Vol. 249, 1959, p. 348-351.
17. Mazelsky, B., Extension of Power Spectral Methods of Generalized Harmonic Analysis to Determine Non-Gaussian Probability Functions of Random Input Disturbances, and Output Responses of Linear Systems, Journal of the Aeronautical Sciences, Vol. 21, No. 3, March 1954.

END